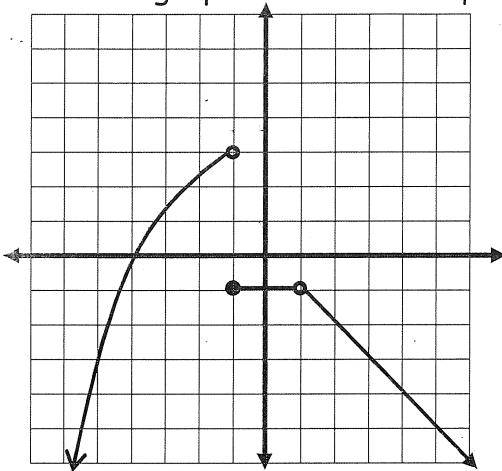


Calculus I

Chapter 2 Test Review

Use the graph to find the requested values.



1) $\lim_{x \rightarrow 1^-} f(x) = 3$

2) $\lim_{x \rightarrow 4} f(x) = 0$

3) $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

4) $\lim_{x \rightarrow \infty} f(x) = -\infty$

5) $\lim_{x \rightarrow -\infty} f(x) = -\infty$

6) $f(1) = \text{DNE}$

7) $\lim_{x \rightarrow 3^+} f(x) = -3$

8) $f(-4) = 0$

9) $f(-1) = -1$

10) $\lim_{x \rightarrow 0} f(x) = -1$

Using the given piecewise function, find the requested values and justify your answers.

$$f(x) = \begin{cases} x^2 - 6x - 2 & \text{if } x \leq -1 \\ \frac{x^2 + 4}{x + 2} & \text{if } -1 < x \leq 3 \\ -2x + 5 & \text{if } x > 3 \end{cases}$$

11) $\lim_{x \rightarrow -1^-} f(x) = 5$

12) $\lim_{x \rightarrow -1^+} f(x) = 5$

13) $\lim_{x \rightarrow 1} f(x) = 5$

14) $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

15) $f(2) = 2$

16) $f(4) = -3$

17) $\lim_{x \rightarrow 4.5} f(x) = -4$

18) $f(-1) = 5$

19) $f(3) = 13/5$

20) $\lim_{x \rightarrow \infty} f(x) = -\infty$

Find the following limits.

21) $\lim_{x \rightarrow \infty} \frac{x^2 - 2x - 8}{x - 4} = \infty$

22) $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x - 4} = 6$

23) $\lim_{x \rightarrow 3} \frac{5x - 9}{x^2 - 5} = \frac{6}{4} = \frac{3}{2}$

24) $\lim_{x \rightarrow \infty} \frac{2 - 6x - 3x^2}{2x^3 + 8x - 2} = 0$

25) $\lim_{x \rightarrow 2} \frac{5x}{x^2 + 3x - 10} = \text{DNE}$

26) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

27) $\lim_{x \rightarrow 1} \frac{x^2 + 9x - 10}{x^3 - 1} = \frac{11}{3}$

28) $\lim_{x \rightarrow \infty} \frac{4x^3 - 6x^6 - 6}{5x^6 + 3x^3 - 5x^2} = -\frac{6}{5}$

29) $\lim_{x \rightarrow \infty} \frac{x - 6}{\sqrt{2x^2 + 5}} = \frac{1}{\sqrt{2}}$

~~(x+1)(x+10)~~

~~(x+1)(x^2+x+1)~~

30) $\lim_{x \rightarrow 1} \frac{x}{|x - 1|} = \infty$

31) $\lim_{x \rightarrow 3} 9 = 9$

32) $\lim_{x \rightarrow -\infty} 4x - 6 = -\infty$

$$\frac{1.1}{.1} = \frac{9}{1}$$

Determine if the following functions are continuous or not. If not, state its type discontinuity and where it occurs.

$$33) f(x) = \frac{x^3 + 27}{x + 3}$$

$$\frac{(x+3)(x^2 - 3x + 9)}{(x+3)}$$

DISC. @ $x = -3$

$$34) f(x) = \frac{x-6}{x^2 - x - 6}$$

$$(x-3)(x+2)$$

$$x = 3, -2$$

DISC. @ $x = 3, -2$

$$35) f(x) = \frac{2x-6}{|x-3|}$$

DISC @ $x = 3$

Determine whether the following piecewise function is continuous. Show all work that leads to your decision. (A sketch may be helpful in determining your answer)

$$36) f(x) = \begin{cases} -3 & x \leq -1 \\ x^3 & -1 < x < 2 \\ 2x + 4 & x \geq 2 \end{cases}$$

$$f(-1^-) = -3 \quad f(2^-) = 2^3 = 8$$

$$f(-1^+) = -1 \quad f(2^+) = 2(2) + 4 = 8$$

$$37) f(x) = \begin{cases} \frac{1}{x-4} & x < 3 \\ 2x-7 & x \geq 3 \end{cases}$$

$$f(3^-) = \frac{1}{3-4} = -1$$

$$f(3^+) = 2(3) - 7 = -1$$

For each of the following, find the value of 'a' that will make $f(x)$ continuous for all values of x .

$$38) f(x) = \begin{cases} ax + 1 & x < 2 \\ a + \sqrt{x+14} & x \geq 2 \end{cases}$$

$$2a + 1 = a + \sqrt{16}$$

$$2a + 1 = a + 4$$

$$\boxed{a = 3}$$

$$39) f(x) = \begin{cases} ax^2 - 2 & x \leq -6 \\ -5x - 8 & x > -6 \end{cases}$$

$$a(-6)^2 - 2 = -5(-6) - 8$$

$$36a - 2 = 30 - 8$$

$$36a = 24$$

$$\boxed{a = 2/3}$$

40) Graph a function that meets the following criteria. Assume the graph is continuous unless contradicted by the criteria.

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = 2$$

$$\lim_{x \rightarrow 3^+} f(x) = \infty$$

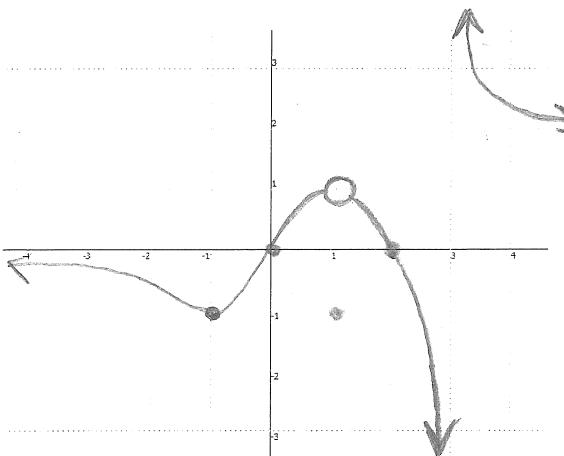
$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$f(-1) = -1 \quad f(0) = 0 \quad f(1) = -1$$

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$f(3) = \text{DNE}$$

$$f(2) = 0$$



Calculus I
Chapter 2 Review

Find each limit or state if it does not exist.

1. $\lim_{x \rightarrow 2} x^3(x-4)$

$$\begin{aligned} & (-2)^3(-2-4) \\ & \boxed{48} \end{aligned}$$

2. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

$$\begin{aligned} & \frac{(-2)^2 - (-2) - 2}{-2 - 2} \\ & \boxed{-1} \end{aligned}$$

3. $\lim_{x \rightarrow \infty} \frac{1}{x^2 - 7x + 1}$

$$\boxed{0}$$

4. $\lim_{x \rightarrow 5^+} \frac{1}{5-x}$

$$\begin{aligned} \frac{1}{5-5.1} &= \frac{1}{-0.1} \rightarrow \infty \\ \boxed{\infty} \end{aligned}$$

5. $\lim_{x \rightarrow \infty} 2 + \frac{1}{x} + \frac{3}{x^2}$

$$\boxed{2}$$

6. $\lim_{x \rightarrow 3} 4$

$$\boxed{4}$$

7. $\lim_{x \rightarrow 5} \frac{x}{5-x}$

$$\frac{5.1}{5-5.1} = \frac{5.1}{-1} \rightarrow \infty$$

$$\frac{4.9}{5-4.9} = \frac{4.9}{-1} \rightarrow -\infty$$

$$\boxed{\text{DNE}}$$

8. $\lim_{x \rightarrow 0} \frac{x^3 - 4x + \sin x}{2x}$

$$= \frac{x^3}{2x} - \frac{4x}{2x} + \frac{\sin x}{2x}$$

$$= \frac{x^2}{2} - \frac{4}{2} + \cancel{\frac{\sin x}{2x}}$$

$$= 0 - 2 + \frac{1}{2} = \boxed{-\frac{3}{2}}$$

9. $\lim_{h \rightarrow 0} \frac{1}{\sqrt{4h+3} + \sqrt{3}}$

$$\begin{aligned} & \frac{1}{\sqrt{4 \cdot 0 + 3} + \sqrt{3}} \\ & \frac{1}{\sqrt{3} + \sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}} \end{aligned}$$

10. $\lim_{y \rightarrow 0} \frac{\sin 3y}{y}$

$$\boxed{3}$$

11. $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 1}}{x - 1} = \frac{\sqrt{2} \cdot \sqrt{x}}{x}$

$$\boxed{-\sqrt{2}}$$

12. $\lim_{x \rightarrow \infty} \frac{-x^4 + x^3}{21x^3 + 32}$

$$\lim_{x \rightarrow \infty} \frac{-x}{21} \rightarrow \boxed{-\infty}$$

13. $\lim_{x \rightarrow \infty} \frac{4x^2 + 5}{7x^2 - 2}$

$$\boxed{\frac{4}{7}}$$

14. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - 2x^2 + x}$

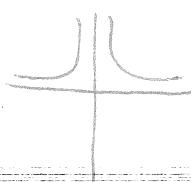
$$\cancel{(x-1)(x-1)}$$

$$x(x-1)(x-1)$$

$$\boxed{1}$$

15. $\lim_{x \rightarrow 0^-} \frac{1}{x^2}$

$$\frac{1}{(0^-)^2} = +\infty$$



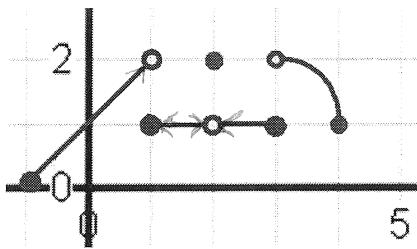
16. $\lim_{x \rightarrow \infty} \sqrt{7x^2 + 2} - x$

$$\begin{aligned} & \sqrt{7x^2 + 2} - x \\ & \sqrt{7x^2 + 2} + x \end{aligned}$$

$$\begin{aligned} & \frac{7x^2 + 2 - x^2}{\sqrt{7x^2 + 2} + x} = \frac{6x^2}{\sqrt{7x^2 + 2} + x} \rightarrow 00 \\ & \boxed{00} \end{aligned}$$

$$\boxed{00}$$

Evaluate each limit for the given graph.



17. $\lim_{x \rightarrow 2} f(x)$

18. $\lim_{x \rightarrow 4} f(x)$

19. $\lim_{x \rightarrow 1} f(x)$

20. $\lim_{x \rightarrow 3^-} f(x)$

21. $\lim_{x \rightarrow 3^+} f(x)$

22. $f(3)$

23. $f(2)$

24. $\lim_{x \rightarrow -1^+} f(x)$

For the graph above, determine if the given intervals are continuous.

25. $[-1, 1]$

NO

26. $[-1, 1]$

YES

27. $(1, 2)$

YES

28. $(1, 2]$

NO

29. Find the points of discontinuity for the graph above on the interval $[-1, 4]$.

1, 2, 3

31. Find the value of k that will make the function continuous: $f(x) = \begin{cases} \frac{x^2 + 2x - 15}{x - 3}, & x \neq 3 \\ k & x = 3 \end{cases}$

$$\cancel{(x-3)(x+5)} = k \quad @ \quad x=3$$

$$3+5=k$$

32. Find any discontinuities for the function. $f(x) = \frac{x-3}{x^2-9}$

$$(x-3)(x+3)$$

$x = 3, -3$
 ↓ ↓
 HOLE V.A.